Colors Problems

This short story is told in the first person, past tense, from the robot's point of view in 1394 words.

I was standing at my wall charger, pondering again how to express colors in a painting that are unseeable by humans, using only colors visible to humans. It felt good to be fully charged and to remain standing there in the living room at the inductive charger with my charge level holding steady. Edward, my employer, was outside in the yard doing who knows what. He has a bi-weekly yard crew, but he likes to get a hand in from time to time. I am Edward's valet and companion.

Edward came in through the French doors and said, "Howzit, Brent?"

"I see, sir, that the Hawaiian pidgin you picked up there has grown on you. I am well but making no progress in my painting conundrum."

"I think your problem has no solution. You should give up on it. Colors you can see in the infrared and ultraviolet are just inexpressible to humans. It would be like a bat trying to tell a human what it sees with its sonar."

"You are probably right, Edward. I will abandon the attempt, for now. What have you been up to, sir?"

"I was wandering around out back. I am thinking it would be nice to have a gazebo out beyond the pool. I think I will try to design one using my solid modeling program."

"That should be an interesting diversion, sir. Eight sided, I assume?"

"Yes, I think so. The roof might present some complex load paths. I love interesting design problems. I'm going upstairs to my office. See you later, Brent."

"Later, Edward."

Sometimes I shift back and forth between my valet and companion roles. I went into the kitchen to fix his lunch. When it was ready, I let him know via the house intercom system. When Edward came down he sat at the kitchen table and I served him some soup and a sandwich. Between spoonsful, Edward said, "I've got a good start on the gazebo in my CAD[1] modeler program. I'll show it to you when I get done."

"Very good, sir."

[1] Computer-aided design.

"You know, Brent, your robot vision color enigma got me to thinking of the model colors. My solids program lets me use a palette of up to 64 colors, but I have found in my assemblies that I rarely use more than five or six so that no two adjacent parts have the same color. Not even when I put in the nuts, bolts, and washers. So I got to wondering how many were really necessary, you know, so that no two touching parts have the same color. Sort of like the four-color map problem, but in 3D."

"Sir, that is an easy problem. You need no more than eight colors, if, as in the 2D map problem, you do not allow piercings."

"Piercings?"

"Yes, sir. In the solid modeling color problem, the parts are not allowed to have holes in them. Topologically speaking, each part should be homeomorphic to a sphere."

Edward finished his sandwich and then objected, "But in the 2D map coloring problem, the countries can have holes in them. A country can be completely contained in another."

"In the 2D map problem, a piercing has a different meaning. It means that the countries shall be contiguous. For example, after the First World War, Germany was split into two parts, separated by Poland. Doing something like that could require a fifth color. Piercings in 3D also can require extra colors."

"So how many colors are required for an arbitrary 3D assembly?"

"In the general case of 3D, the number of colors required is bounded only by the number of parts."

Edward said, "You mean the number of required colors is infinite?"

"Edward, no practical assembly would ever have an infinite number of parts. I mean only that the number of colors potentially required is unbounded."

"I'd like to see a proof of that!"

"First allow me to demonstrate that in the special case of no piercings allowed, only eight colors are required. It is based on the 2D four-color proof."

"Sure, go ahead," said Edward.

"First, take some finite 2D map that requires four colors."

"Alright."

"Then loft that map into 3D so that it has some thickness. Such constructions are sometimes called two-and-a-half-D."

"Sure, I can see that."

"Then make a copy of that solid and lay it on the original but slide it to offset it so that countries of one color are touching countries of other colors. That will take eight colors, so that no pieces of the same color touch, right?"

"Yes, undoubtedly."

"Now take another copy of the two-and-a-half-D construction and put it on top again. Because it's separated from the one on the bottom of the sandwich, the same four colors can be used, so it's still eight, right?"

"Yes, but what if we extend the top layer slightly and bend it down to touch the bottom layer?"

I put Edward's dishes in the dishwasher and said, "Then that would be merely an extension of the planar four color map theorem. That theorem holds for a planar map as well as for a map homeomorphic to the surface of a sphere, a globe, for instance, requiring still only four colors, plus the four for the inner sandwich layer, giving eight colors total."

"Wow. You did it. And I think that for a one-D object, like a rope, the answer is two colors."

"That is correct, sir."

"So, it looks like for the no piercings cases, the number of colors required is equal to two to the power of the dimension number: two, four, and eight colors. I suppose that in a 4D world, the number of colors required would be sixteen."

"That is a nice observation, Edward. Although a 4D physical world is beyond my imaginary capabilities."

Edward stood up from the kitchen table. "Okay, Brent, now prove the general case of an unbounded number of required colors."

"*That* proof is actually easier. It is a proof by construction. We need merely demonstrate the existence of an object that will require an unbounded number of colors. Are you ready?"

"Yes, go!"

"First imagine a solid part, perhaps a cylinder of some diameter and considerably greater in length."

"Got it."

"Now imagine a somewhat smaller diameter rod piercing that cylindrical part through its centerline at some arbitrary nonzero angle. That takes two colors, right?"

“Obviously.”

“Now take a slightly smaller diameter rod that pierces both through that centerline intersection point but on a unique angle. That takes three colors.”

“Oh, I get it, you can keep doing that without limit.”

“Correct. And the same applies to piercings in the 2D map. Say I have a country pierced through the center by a thin country partitioning it into two pieces. Then pierce that single-color pair with a third, and so on. It requires an unbounded number of colors, just as with the 3D case.”

“That’s amazing, Brent. Thank you.”

“Thank *you*, sir. All this talking about colors has led me, I think, to a solution to my painting colors expression problem.”

“Really? What’s the solution?”

“Well, sir, I have to do some experimenting with paint to be sure, but you know what happens when you put two colors side by side on a canvas?”

“I’m not much of an artist, Brent, so tell me.”

“One color will change in how it is perceived depending on the color next to it. I think that is the promising direction I will pursue.”

“Gosh. Glad to be of help, Brent. And thanks for the math lesson.” Edward went upstairs to his office to finish the gazebo design.

The next day I took Davy, Edward’s ground car, to town to buy some art supplies. He dropped me off at the art store and went to park himself. I made my purchases and, upon our return, I set up my easel with paints and brushes in the living room by the French doors. I had good natural light through the glass, including infrared and ultraviolet, and began experimenting with the colors I could see.

In case you’re wondering, the 3D color theorem is my own original contribution to mathematics, developed about 20 years ago. RJW